How to build high-quality filters out of low-quality parts

If the operational amplifier is wired as a voltage follower, an active filter can use loose-tolerance resistors and capacitors—and still perform well

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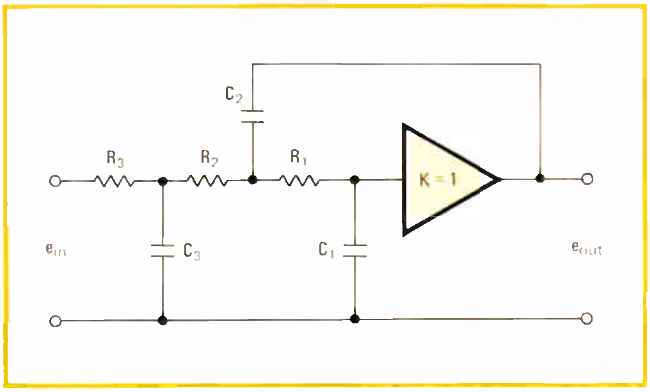
☐ An active filter's sensitivity to component values is an important factor in determining its cost. If the circuit is designed for minimum sensitivity to these values, then inexpensive low-tolerance components can be used without harm to its performance.

In the three-pole filter of Fig. 1, the amplifier closed-loop gain, K, is the parameter that governs sensitivity. This results from the presence of the factor $(1 - K)$ in terms of the transfer-function denominator. If K is greater than unity, the denominator will contain negative terms that lead to high sensitivity. For this reason, the amplification is provided by an operational amplifier connected as a voltage follower. In addition to providing $K = 1$, use of a voltage follower also saves two gain-fixing resistors in the op-amp connection.

Low-pass design

If the amplifier in Fig. 1 is a voltage follower and the resistor values are all 1 ohm, a normalized Butterworth or Chebyshev filter response is obtained by choosing capacitance values from Table 1. The zero-ripple design is the normalized Butterworth filter. It has an attenuation of 3 decibels at a frequency of 1 radian per second, and its asymptotic slope in the stopband is 18 dB per octave. The exact attenuation at any frequency ω is given by the formula:

$$e_{out}/e_{in} = 1/(1+\omega^6)^{1/2}$$



1. Uses loose-tolerance components. The performance of this three-pole low-pass active filter is insensitive to values of resistors and capacitors if the amplifier gain, K, is unity. Therefore inexpensive components with relatively loose tolerances can be used.

TABLE 1: ELEMENT VALUES FOR NORMALIZED THREE-POLE ACTIVE FILTERS

Ripple (dB)	C_1	C_2	C_3
0	0.20245	3.5468	1.3926
0.01	0.091294	2.5031	0.84044
0.03	0.097357	3.3128	1.0325
0.10	0.096911	4.7921	1.3145
0.30	0.085819	7.4077	1.6827
1.00	0.05872	14.784	2.3444

TABLE 2: ATTENUATION CONSTANTS FOR CHEBYSHEV FILTERS

Ripple (dB)	Ripple factor (ϵ^2)
0.01	0.00230524
0.03	0.00693167
0.1	0.023293
0.3	0.0715193
1.0	0.258925

TABLE 3: BAND-EDGE FREQUENCY, ω_C, AS A FUNCTION OF RIPPLE AND BAND-EDGE ATTENUATION α

Ripple (dB)	Band-edge frequency ω_C (rad/s)		
	α = 1 dB	α = 2 dB	α = 3 dB
0	0.798355	0.914491	1.00000
0.01	1.56352	1.74229	1.87718
0.03	1.36673	1.50770	1.61524
0.10	1.20154	1.30707	1.38899
0.30	1.08934	1.16726	1.22906
1.00	1.00000	1.05219	1.09487

The capacitance values corresponding to the nonzero values of ripple in Table 1 yield Chebyshev filters. These filters have equiripple passbands, with the edge of the ripple band at 1 rad/s. The attenuation for the Chebyshev filter designs can be calculated from the voltage-ratio formula:

$$e_{out}/e_{in} = 1/[1 + \epsilon^2(4\omega^3 - 3\omega)^2]^{1/2}$$

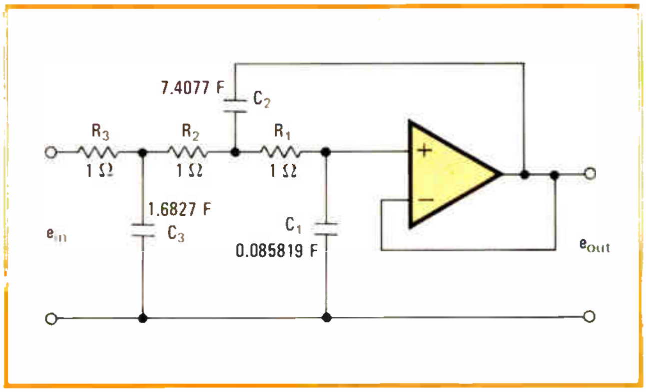
where ϵ^2 is the ripple factor obtained from Table 2. The user can be expected to define the attenuation at the edge of the passband, but the choice of ripple value is usually left to the designer. As an aid to meeting the passband edge requirement, Table 3 gives the frequencies for attenuations of 1, 2, and 3 dB for all ripple values.

As an example, consider the design of a low-pass Chebyshev filter with 0.3-dB ripple and the calculation of its attenuation at twice the 3-dB frequency.

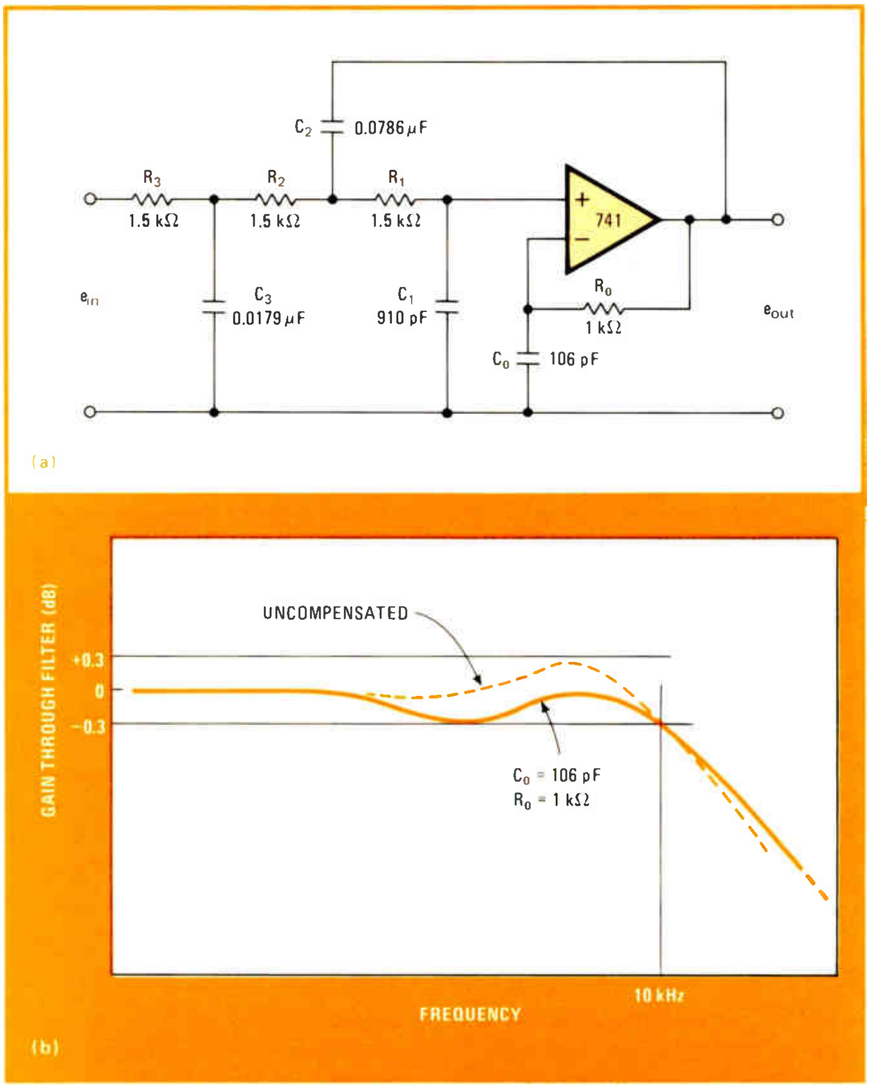
Frequency scaling

The normalized circuit is shown in Fig. 2, with capacitance values taken from Table 1. Table 2 shows that a 0.3-dB ripple corresponds to ϵ^2 of 0.0715193. The 3-dB frequency of this filter is given by Table 3 as 1.22906 rad/s; at twice this frequency, i.e., ω = 2.45812 rad/s, the attenuation is calculated to be 0.0716734. Expressed in decibels, this attenuation is 20 log (0.0716734), or −22.89 dB.

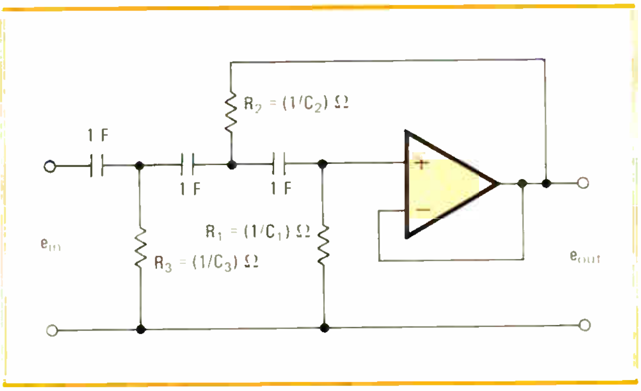
To scale the design so that the edge of the ripple band (that is, the highest 0.3-dB frequency) occurs at 10 kilohertz, divide all capacitance values by $2\pi(10{,}000)$.



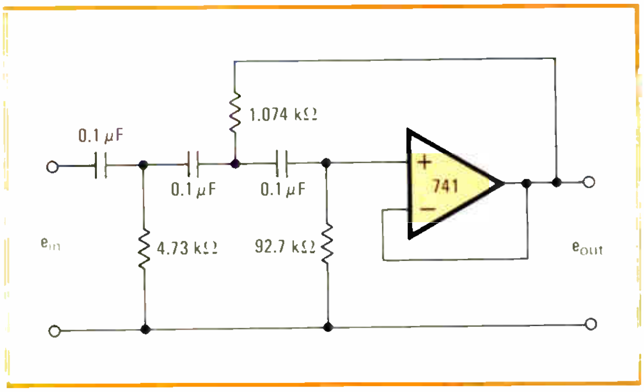
2. Normalized low-pass filter. Design of low-pass Chebyshev filter with 0.3-dB ripple starts with R values of 1 ohm and C values from Table 1, plus a voltage-follower op amp to make K = 1.



3. Practical 10-kHz low-pass filter. The example in the text shows how the component values in Fig. 2 are scaled to provide the edge of the ripple band (the highest frequency at which attenuation is equal to or less than 0.3 dB) at 10 kHz. Boctor compensation is added to voltage follower that uses a type 741 operational amplifier.



4. High-pass filter. In the normalized high-pass three-pole active filter, the series capacitors are all 1 F, and the resistor values (in ohms) are found by reciprocating the numbers in Table 1.



5. Practical 200-Hz high-pass filter. The example in the text gives details of the design of this filter, which has 0.3-decibel ripple above 200 hertz and 31.35-dB attenuation at 60 Hz.

This gives:

$C_1 = 1.36585\ \mu F$
$C_2 = 117.897\ \mu F$
$C_3 = 26.7888\ \mu F$

These values are correct but impractical. A practical design can be obtained by multiplying all impedances by a suitable scale factor, say 1,500. This gives:

$R_1 = R_2 = R_3 = 1{,}500\ \Omega$
$C_1 = 910.6\ pF$
$C_2 = 0.0786\ \mu F$
$C_3 = 0.0179\ \mu F$

The frequency response of this filter is the same as that of the normalized design, except that all attenuations are referred to the band-edge frequency, f_c, of 10 kHz, instead of to 1 rad/s. Consequently, the attenuation formulas serve for the denormalized filters if ω is replaced by f/f_c.

Gain compensation

These filters require rather precise values of closed-loop gain, which is why an operational amplifier is used instead of some simpler unity-gain amplifier. But even an op amp may require compensation if its open-loop gain drops to less than 60 dB at the band edge of the filter.

A voltage divider consisting of resistor R_o and capacitor C_o provides the Boctor compensation, as it is called, by shunting off part of the feedback at high frequencies. R_o and C_o are given by:

$$R_oC_o = 1/(2.6 \times 2\pi f_o)$$

where f_o is the 0-dB frequency of the amplifier open-loop gain curve.

For a 741 op amp, the open-loop gain is less than 40 dB at 10 kHz, and the measured value of f_o is 581 kHz. Therefore, R_o and C_o are 1 kilohm and 106 picofarads, respectively. The complete circuit for the 10-kHz filter in the example, including Boctor compensation, is shown in Fig. 3.

High-pass design

The element values given in Table 1 are the reciprocals of the resistances in a normalized high-pass filter, as shown in Fig. 4. The normalized capacitance values for the high-pass filter are all 1 farad. In the attenuation formula used for frequency-response calculations, ω must be replaced by f_c/f, where f_c is the band-edge frequency; this applies to both normalized and denormalized highpass filters.

As an example, consider the design of a high-pass filter that must pass signals above 200 hertz and must suppress 60-Hz signals by at least 30 dB. Here f_c/f is 200/60; the voltage-ratio expression shows that a Chebyshev filter with 0.3-dB ripple will give an attenuation of 31.35 dB. Therefore, the resistors in the normalized circuit (Fig. 4) have values found from Table 1 as follows:

$R_1 = 1/C_1 = 1/0.085819 = 11.6524\ \Omega$
$R_2 = 1/C_2 = 1/7.4077 = 0.1350\ \Omega$
$R_3 = 1/C_3 = 1/1.6827 = 0.59428\ \Omega$

Frequency-scaling this design to $\omega_c = 2\pi(200\ Hz)$ gives 1256.6 for ω_c, and dividing the 1-F capacitances by this number gives the value $C_1 = C_2 = C_3 = 795.8\ \mu F$ for the capacitors.

Finally, to get practical component values, all the impedances are multiplied by the factor 7,958 to make the capacitors 0.1 μF. The resistors then turn out to be as shown in Fig. 5. □

How to design three-pole filters

The following equations will yield component values for three-pole active filters not covered by Table 1. Given:

$$e_{out}/e_{in} = 1/(a_3s^3 + a_2s^2 + a_1s + 1)$$

with known numerical coefficients a_3, a_2, a_1, component values for the circuit in Fig. 1 are obtained by finding a positive real root of:

$$18x^3 - 12a_1x^2 + (2a_1^2 + 3a_2)x + (2a_3 - a_1a_2) = 0$$

If x_0 is such a root, then:

$$y = -a_3/(3x_0^2 - a_1x_0)$$
$$z = a_1 - 3x_0$$

Then the element values for Fig. 1 are $K = 1$, $R_1 = R_2 = R_3 = 1$, $C_1 = x_0$, $C_2 = y$, and $C_3 = z$, where resistances are in ohms and capacitances are in farads.